N-FIBER-FULL MODULES

HONGMIAO YU

At the "CIME-CIRM Course on Recent Developments in Commutative Algebra" conference in 2019, Matteo Varbaro introduced the notion of "fiber-full modules" providing a new proof of the main result of [1]. The starting point of studing N-fiber-full modules is to find some possible generalizations of this concept.
Suppose that A is a Noetherian flat $K[t]$-algebra, M and N are finitely generated A-modules which are flat over $K[t]$, and all of A, M and N are graded $K[t]$-modules. Varbaro showed in his talk that if M is fiber-full, then $\operatorname{Ext}^i_A(M,A)$ is flat over $K[t]$ for all $i \in \mathbb{Z}$. We introduced the "N-fiber-full up to h" modules and we considered the following question: if M is N-fiber-full up to h as an A-module, can we obtain the flatness of some $\operatorname{Ext}^i_A(M,N)$? In the first part of this talk we will see that

MainTheorem. *Let h be an integer. M is N-fiber-full up to h as an A-module if and only if $\operatorname{Ext}^i_A(M,N)$ is flat over $K[t]$ for all $i \leq h-1$.*

After that we will see some applications of this theorem. A main consequence is that the notion "N-fiber-full up to h" allows us to infer interesting results whenever the special fiber M/tM has "nice" properties after removing primary components of big height. For example, we will see the following Theorem:

Theorem. *Let S be the polynomial ring $K[X_1,\dots,X_n]$ over a field K, let $I \subseteq S$ be a homogeneous ideal. Fixed a monomial order on S, we denote by $\operatorname{in}(I)$ the initial ideal of I with respect to this monomial order. If I is such that $\operatorname{in}(I)^{\mathrm{sat}}$ is square-free, then*

$$\dim_K H^i_{\mathfrak{m}}(S/I)_j = \dim_K H^i_{\mathfrak{m}}(S/\mathrm{in})_j$$

for all $i \geq 2$ and for all $j \in \mathbb{Z}$.

Another interesting observation is: if S is $K[t]$-fiber-full, then the graded Betti numbers are preserved going from I to $\operatorname{in}(I)$.

References

[1] Aldo Conca; Matteo Varbaro. Square-free Gröbner Degenerations, *Invent. Math.* Volume **221**, Issue **3**, 1 September 2020, 713-730.

University of Genoa
Email address: yu@dima.unige.it