

K-STABILITY AND THE HITCHIN-CSCK SYSTEM

CARLO SCARPA

One of the most important and fundamental results in complex geometry is the Hitchin-Kobayashi correspondence on vector bundles, which links the stability of a holomorphic vector bundle with the existence of Hermite-Einstein connections on the bundle. This correspondence can be seen as an infinite-dimensional analogue of the Kempf-Ness Theorem, which relates the existence of solutions to a moment map equation to GIT-stability. This point of view has inspired a great deal of research in complex geometry, as many important structures of compact complex manifolds can be interpreted as the existence of solutions of some (infinite-dimensional) moment map equation.

Of particular interest is the existence of Kähler metrics with special curvature properties on compact Kähler manifolds, such as Kähler-Einstein metrics, constant scalar curvature (cscK) or extremal Kähler metrics. Inspired by the case of Hermite-Einstein metrics, it has been conjectured that the existence of cscK metrics should be equivalent to an algebraic condition called K -stability. Our goal is to extend this parallel between the Hermite-Einstein and the cscK problems by studying, on the category of polarized varieties, a system of equations analogous to Hitchin's Higgs bundle equations on vector bundles.

The system is obtained from an infinite-dimensional hyperkähler reduction, which gives a set of moment map equations for a Kähler metric and infinitesimal deformation of the complex structure, which plays the role of a Higgs field in the category of Kähler manifolds. The system reduces to the cscK equation when the infinitesimal deformation vanishes. This approach is based on an explicit construction of hyperkähler metrics by Biquard and Gauduchon and generalizes a previous result by Donaldson on complex curves. We formulate a generalization of K -stability that should characterize the existence of solutions to this system, and we will sketch the proof of this conjecture for toric manifolds. Based on joint work with Jacopo Stoppa.