VARIATIONAL CONVERGENCES FOR FUNCTIONALS AND DIFFERENTIAL OPERATORS DEPENDING ON VECTOR FIELDS

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Let $\Omega \subset \mathbb{R}^n$ be a bounded domain. The X-gradient is a family of Lipschitz continuous vector fields $X = (X_1, .., X_m)$ $(m \leq n)$ that are pointwise linearly independent, outside Lebesgue measure zero sets. The Sobolev spaces associated with the X-gradient are

$$W_X^{1,p}(\Omega) := \{u \in L^p(\Omega) : X_j u \in L^p(\Omega) \text{ for } j = 1, \dots, m\}$$

and $W_{X,0}^{1,p}(\Omega)$, defined as the closure of $\mathbf{C}_c^1(\Omega) \cap W_X^{1,p}(\Omega)$ in $W_X^{1,p}(\Omega)$. We are interested in families X satisfying a global Poincaré inequality and such that $W_{X,0}^{1,p}(\Omega)$ compactly embeds into $L^p(\Omega)$, $p \in [1, \infty)$. For such families, the following Γ-compactness result will be showed.

Theorem 1. *Let $1 < p < \infty$ and define the sequence $F_h : L^p(\Omega) \to \mathbb{R} \cup \{\infty\}$, $h \in \mathbb{N}$, by*

$$F_h(u) := \begin{cases} \int_\Omega f_h(x, Xu(x))dx & \text{if } u \in W_X^{1,p}(\Omega) \\ +\infty & \text{if } u \in L^p(\Omega) \setminus W_X^{1,p}(\Omega) \end{cases},$$

where $f_h : \Omega \times \mathbb{R}^m \to \mathbb{R}$ is a Carathéodory function, convex w.r.t. the second variable, satisfying

$$c_0|\eta|^p - a_0(x) \leq f(x, \eta) \leq c_1|\eta|^p + a_1(x) \quad \text{for a.e. } x \in \Omega \text{ for any } \eta \in \mathbb{R}^m,$$

with $a_0, a_1 \in L^1(\Omega)$ nonnegative and $c_0 \leq c_1$ positive constants, and Borel-measurable on Ω.

Then, there exist $f : \Omega \times \mathbb{R}^m \to \mathbb{R}$, satisfying the same hypotheses of f_h (with the same constants and L^1 functions) and $F : L^p(\Omega) \to \mathbb{R} \cup \{\infty\}$ such that (up to subsequences)

1) *F_h Γ-converges to F in the strong topology of $L^p(\Omega)$, as $h \to \infty$;*
2) *the limit F can be represented by*

$$F(u) := \begin{cases} \int_\Omega f(x, Xu(x))dx & \text{if } u \in W_X^{1,p}(\Omega) \\ +\infty & \text{if } u \in L^p(\Omega) \setminus W_X^{1,p}(\Omega) \end{cases}.$$

As a consequence of the previous result, we show that the class of linear differential operators in X-divergence form is closed in the topology of the H-convergence, by adapting a variational approach introduced by Ansini, Dal Maso and Zeppieri [1].
This is a joint work with Andrea Pinamonti, Francesco Serra Cassano (University of Trento) [4, 5], Fabio Paronetto (University of Padova) and Eugenio Vecchi (University of Bologna) [3].

References

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