

# GENERALIZED HEEGNER CYCLES AND $p$ -ADIC $L$ -FUNCTIONS IN A QUATERNIONIC SETTING

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A central topic in algebraic number theory is the theory of Galois representations, continuous group homomorphisms from a Galois group to the group of automorphisms of a vector space. To a suitable Galois representation one can attach two fundamental objects: an  $L$ -function and a Selmer group.  $L$ -functions are particular complex functions which have a special role in number theory. They arise in various situations: one can attach an  $L$ -function also to a number field, an elliptic curve or more in general an abelian variety, a modular form, a Dirichlet or a Hecke character. A typical example is the famous Riemann zeta function, which is known to contain information about the prime numbers. More in general,  $L$ -functions are expected to encode a lot of information on the arithmetic objects they are attached to. The *Selmer group* is instead an algebraic object, which is an invariant of the representation that contains deep number theoretical information about it. Even though these two objects have really different natures, they are related by the *Bloch-Kato Conjecture* which, in the cases of our interest, predicts an equality between the order of vanishing of the  $L$ -function at a certain point and the dimension of the Selmer group.

In [3], Castella and Hsieh proved significant results about Selmer groups associated with Galois representations attached to newforms (suitable modular forms) twisted by Hecke characters of an imaginary quadratic field. These results lead also to prove other instances of the rank 0 case of the Bloch–Kato Conjecture for  $L$ -functions of modular forms. The key point of the work of Castella and Hsieh is a remarkable link between certain arithmetic objects called *generalized Heegner cycles* that were introduced by Bertolini, Darmon and Prasanna in [1] and suitably defined  $p$ -adic  $L$ -functions, which are instead objects of  $p$ -adic analytic nature, interpolating special values of complex  $L$ -series.

All these results are obtained under the so-called *Heegner hypothesis* that the imaginary quadratic field must satisfy with respect to the level of the modular form. What happens if one weakens the Heegner hypothesis, considering more quadratic fields? In this talk, we answer to that question and see that several of the results of Castella–Hsieh can be extended to a quaternionic setting, which is the setting that arises when one works under a “relaxed” Heegner hypothesis. This can be done working with *generalized Heegner cycles* over Shimura curves (instead of modular curves) introduced by Brooks in [2]. For all the details, see [4].

## REFERENCES

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- [3] F. CASTELLA, M. HSIEH, *Heegner cycles and  $p$ -adic  $L$ -functions*, Math. Ann. **370**, pp. 567–628, 2018.
- [4] P. MAGRONE, *Generalized Heegner cycles and  $p$ -adic  $L$ -functions in a quaternionic setting*, to appear in Ann. Sc. Norm. Super. Pisa Cl. Sci., arXiv:2008.13500.