EQUIVARIANT LOCALIZATION METHODS, ORIENTATIONS AND MODULARITY

EUGENIO LANDI

Atiyah, in [1], using ζ-regularization techniques and the equivariant localization theorem, recovered the $\hat{A}$-genus of a Spin manifold as a class in equivariant cohomology. In particular he showed that (roughly)

$$\hat{A}(M) = \int_M \frac{1}{\mathrm{eul}(\nu)}$$

where ν is the (infinite rank, S^1-equivariant) normal bundle of $\iota : M \to \mathcal{L}M = \mathrm{Maps}(S^1, M)$ and $\mathrm{eul}(\nu)$ is its Euler class.

In [2], we generalize this fact to the space of maps from elliptic curves and to the Witten genus. In particular, introducing a suitable "anti-holomorphic sector" we prove a generalization of the equivariant localization theorem. This allows us to consider smooth actions of elliptic curves $\mathbb{C}/\Lambda$ and thus lets us apply ζ-regularization techniques, with rigour not yet found in the literature, to recover the Witten genus as the modular form given (again, roughly), over a point $\tau \in \mathbb{H}$, by the formula

$$\mathrm{Wit}(M)(\tau) = \int_M \frac{1}{\mathrm{eul}(\nu_\tau)}$$

where ν_τ is the (infinite rank, $\mathbb{C}/\Lambda$-equivariant) normal bundle of $\iota : M \to \mathrm{Maps}(\mathbb{C}/\Lambda, M)$, with Λ the lattice generated by 1 and τ, M a rational String manifold, and $\mathrm{eul}(\nu)$ the Euler class of ν.

After a brief introduction of the localization theorem in both the classical and antiholomorphic setting, I will survey some ζ-regularization theory and give precise statements of both Atiyah's result and ours.

This is a joint work with Mattia Coloma and Domenico Fiorenza.

References

[1] M.F. Atiyah, *Circular symmetry and stationary-phase approximation*, Colloque en l'honneur de Laurent Schwartz - Volume 1, pp. 43–59, 1985

[2] M. Coloma, D. Fiorenza, E. Landi, *The (anti-)holomorphic sector in $\mathbb{C}/\Lambda$-equivariant cohomology, and the Witten class*, https://arxiv.org/abs/2106.14945, 2021

Università di Roma Tre

Email address: eugenio.landi@uniroma3.it

URL: https://sites.google.com/view/eugeniolandi/