STRONGLY SINGULAR CONVECTIVE ELLIPTIC EQUATIONS IN $\mathbb{R}^N$ DRIVEN BY A NON-HOMOGENEOUS OPERATOR

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The aim of this talk is to present an existence result for the following problem:

$$\text{(P)} \qquad \begin{cases} -\operatorname{div} a(\nabla u) = f(x,u) + g(x,\nabla u) & \text{in } \mathbb{R}^N, \\ u > 0 & \text{in } \mathbb{R}^N, \end{cases}$$

where $N \geq 2$. The differential operator $u \mapsto \operatorname{div} a(\nabla u)$, usually called a-Laplacian, is patterned after the (p,q)-Laplacian $\Delta_p + \Delta_q$, $1 < q < p < +\infty$. The reaction terms $f : \mathbb{R}^N \times (0,+\infty) \to [0,+\infty)$ and $g : \mathbb{R}^N \times \mathbb{R}^N \to [0,+\infty)$ are Carathéodory functions obeying suitable growth conditions.

Problem (P) possesses at least four interesting peculiarities:

- the operator $u \mapsto a(\nabla u)$ can be non-homogeneous;
- the reaction term f is strongly singular (i.e., it behaves like $u^{-\gamma}$ with $\gamma \geq 1$) and $f(x,\cdot)$ can be non-monotone;
- the reaction term g is convective (i.e., it depends on ∇u);
- the problem is set in the whole space $\mathbb{R}^N$.

Problems exhibiting some of these features arise from applications, in particular in Chemistry and Biology, and have been recently investigated by many authors from different viewpoints, as existence, regularity, and qualitative properties of solutions. To the best of our knowledge, the result presented here represents the first contribution about existence of solutions to quasi-linear singular convective problems in the whole space.

From a technical point of view, several challenges arise in this framework: (i) non-homogeneity of the differential operator prevents to exploit standard procedures in the construction of sub-solutions, and regularity issues do not allow to work directly in the whole space; (ii) due to the strongly singular nature of the problem, gaining compactness from energy estimates requires some efforts, such as localization procedures and fine energy estimates on suitable super-level sets of solutions; (iii) the loss of variational structure compels to use topological and monotonicity methods instead of variational ones; (iv) the setting $\mathbb{R}^N$ causes lack of compactness for Sobolev embeddings.

A variety of techniques will be used to ensure the existence of a generalized solution u (that is, a distributional solution such that $\operatorname{essinf}_K u > 0$ for any compact $K \subseteq \mathbb{R}^N$): in addition to the aforementioned tools, also regularization and approximation procedures (such as the shifting method), sub-super-solution technique, fixed point and regularity theory, maximum and comparison principles, as well as some *ad hoc* compactness results, are employed.

This is a joint work with Laura Gambera.

References

[1] L. Gambera and U. Guarnotta, *Strongly singular convective elliptic equations in $\mathbb{R}^N$ driven by a non-homogeneous operator*, ArXiV preprint no. 2112.08002.

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