SURFACES WITH CANONICAL MAP OF HIGH DEGREE

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Let S be a compact complex surface. If the image of the canonical map ϕ is a surface, then we can consider its *degree* d. Beauville obtained in [1] that $d \leq 9 + \frac{27-9q}{p_g-2}$, where $q = h^{1,0}(S)$ and $p_g = h^{2,0}(S)$ are the Hodge numbers of S. As noted first by Persson, the maximum possible degree is 36 and if $d > 27$ then $q = 0$ and $p_g = 3$.

A question posed by M. Lopes and R. Pardini in [2] is if for each $d \leq 36$ there exists an algebraic surface S such that the degree of its canonical map is equal to d. At the moment there are only examples in literature of surfaces with canonical map of degree $d = 2, \cdots, 9, 12, 16, 20, 24, 27, 32, 36$.

I consider the so called regular product-quotient surfaces, surfaces birational to a quotient $(C_1 \times C_2)/G$, where C_i are curves and G is a finite group acting separately on both factors. The aforementioned results suggest to produce systematically examples of smooth regular product-quotient surfaces with $q = 0$ and $p_g = 3$ following the techniques in [3] and [4] and then to study their canonical maps. There is no general way to compute the canonical map of product-quotient surfaces. However the assumption $p_g = 3$ implies $d = M^2$, where M is the mobile part of the canonical system of the blow-up of S in the base locus of its canonical system. In this case to compute d we have only to describe the intersection of three canonical divisors generating the canonical system $|K_S|$. In this direction I proved the following

Theorem 1. *Let C be a curve, $G < Aut(C)$ be a finite group such that $C/G \cong \mathbb{P}^1$ and let $\pi : C \to \mathbb{P}^1$ be the quotient map. Let $\chi \in Irr(G)$ be an irreducible character of G, call ρ_χ its irreducible representation. Denote by $H^{1,0}(C)^\chi$ the corresponding isotypic component of the induced representation of G on $H^{1,0}(C)$. Call $|K_C|^\chi$ the associated subsystem of the canonical linear system of C given by the isotypic component $H^{1,0}(C)^\chi$. Then the base locus of $|K_C|^\chi$ is*

$$Bs(|K_C|^\chi) = \sum_{q \in Crit(\pi)} (m_q - a_q^\chi - 1)\pi^{-1}(q) \tag{1}$$

where h is the local monodromy of a point $p \in \pi^{-1}(q)$, $m_q := o(h)$ and a_q^χ is defined as

$$a_q^\chi := \max\{\lambda \in [0, \cdots, m_q - 1] : e^{\frac{2\pi i}{o(h)} \cdot \lambda} \text{ is an eigenvalue of } \rho_\chi(h)\}$$

In this talk I will explain how this theorem can be used to investigate the base locus of the canonical system of a product-quotient surface. As application I will give examples of algebraic surfaces with new canonical degrees, 10, 11, 14 and 18, together with two new examples of product-quotient surfaces with $d = 24, 32$.

References

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