# ENTROPY MARTINGALE OPTIMAL TRANSPORT AND NONLINEAR PRICING-HEDGING DUALITY

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The aim of this paper is to develop a duality between a novel Entropy Martingale Optimal Transport problem $(A)$ and an associated optimization problem $(B)$:

$$(A) := \inf_{Q \in \mathrm{Mart}(\Omega)} \left( E_Q\left[c\right] + \mathcal{D}_U(Q) \right) = \sup_{\Delta \in \mathcal{H}} \sup_{\varphi \in \mathbf{\Phi}_\Delta(c)} S^U\left(\varphi\right) =: (B). \tag{1}$$

In $(A)$ we follow the approach taken in the Entropy Optimal Transport (EOT) primal problem by Liero et al. [2] but we add the constraint, typical of the Martingale Optimal Transport (MOT) theory in [1], that the infimum of the cost functional is taken over martingale probability measures, instead of finite positive measures, as in Liero et al. The Problem $(A)$ differs from the corresponding problem in Liero et al. not only by the martingale constraint, but also because we admit less restrictive penalization terms $\mathcal{D}_U$, which may not have a divergence formulation.

In Problem $(B)$, the outer supremum is taken over admissible integrands ($\Delta$, given by continuous functions). Every such integrand models a self-financing trading strategy, with terminal wealth given by the corresponding stochastic integral $I^\Delta$. In the inner supremum of $(B)$, the class $\Phi_\Delta$ consists of static parts of semistatic subhedging strategies for the contingent claim $c$ involving the strategy $\Delta$, namely $\mathbf{\Phi}_\Delta(c) = \{\varphi = [\varphi_0, \dots, \varphi_T] \mid \sum_{t=0}^T \varphi_t(x) + I^\Delta(x) \leq c(x) \ \ \forall\, x \in \Omega\}$. The objective functional $S^U$, associated via Fenchel conjugacy to the term $\mathcal{D}_U$, is not any more linear, as in OT or in MOT. This leads to a novel optimization problem which also has a clear financial interpretation as a nonlinear subhedging value.

Mathematically speaking, the duality we develop is a relaxed version of the classical MOT problem. MOT problems have been widely studied in Mathematical Finance due to the fact that they yield robust bounds for sub/superhedging prices when options can be bought and sold in the market. Historically, it is assumed that the marginals of the pricing measures are known, since these can be inferred from market data. Since in practice these marginals can only be known in an approximate form due to limited data availability, it is also financially relevant to study MOT problems in which marginals are not expected to exactly match the candidate, data-inferred ones. This is among the aims of this work.

Our theory allows us to establish a nonlinear robust pricing-hedging duality, which covers a wide range of known robust results. We also focus on Wasserstein-induced penalizations and we study how the duality is affected by variations in the penalty terms, with a special focus on the convergence of EMOT to the extreme case of MOT.

## References


[1] Beiglböck, Henry-Labordère, Penkner, *Model-independent bounds for option prices—a mass transport approach*, Finance Stoch., 17(3): pp. 477–501, 2013.

[2] Liero, Mielke, Savaré, *Optimal Entropy-Transport problems and a new Hellinger-Kantorovich distance between positive measures*, Inventiones mathematicae, 211(3):pp. 969–1117, 2018.



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