# REFINED GAUSS–GREEN FORMULAS

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The Gauss–Green and integration by parts formulas are of significant relevance in many areas of analysis and mathematical physics. Their importance motivated several investigations to obtain extensions to more general classes of integration domains and weakly differentiable vector fields, thus ultimately leading to the definition of divergence-measure fields. These are $L^p$-summable vector fields on $\mathbb{R}^n$ whose distributional divergence is a Radon measure. It is not difficult to notice that the divergence-measure fields are a generalization of the vector fields of bounded variation.

I shall present the approach to the theory of divergence-measure fields in the Euclidean framework developed in [2], a joint work with Kevin R. Payne. We consider the case $p = \infty$ and prove that the Gauss–Green formulas hold on sets of finite perimeter by suitably manipulating the Leibniz rule for essentially bounded divergence-measure fields and essentially bounded scalar functions of bounded variation. In this way, we define the interior and exterior normal traces of divergence-measure fields and show that they are essentially bounded functions on the reduced boundary of the integration set.

Due to the robustness of the Euclidean theory of divergence-measure fields, we can extend it to some non-Euclidean context. In particular, we develop a theory of divergence-measure fields in noncommutative stratified nilpotent Lie groups, based on [1], a joint work with Valentino Magnani. Since the Euclidean theory relies heavily on De Giorgi's blow-up theorem and related fine properties of functions of bounded variation, which do not hold in general stratified groups, we provide alternative approximation arguments in order to prove a Leibniz rule for essentially bounded horizontal divergence-measure fields and essentially bounded scalar function of bounded h-variation. As a consequence, we achieve the existence of normal traces and the Gauss–Green theorem on sets of finite h-perimeter. Then, we consider some particular cases; that is, fields whose divergence is an absolutely continuous measure and integration over sets with Euclidean finite perimeter.

## References

[1] Giovanni E. Comi, Valentino Magnani, *The Gauss–Green theorem in stratified groups*, Advances in Mathematics, 360: 106916 (2020).

[2] Giovanni E. Comi, Kevin R. Payne, *On locally essentially bounded divergence measure fields and sets of locally finite perimeter*, Advances in Calculus of Variations, 13.2: 179-217 (2020).

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