

LUSIN APPROXIMATION THEOREMS OF ORDER m IN CARNOT GROUPS

MARCO CAPOLLI

Lusin's approximation theorem is a powerful tool used in analysis to approximate non-smooth maps. It asserts that, given a measurable map $f: \mathbb{R}^n \rightarrow \mathbb{R}$, there exists a continuous map $F: \mathbb{R}^n \rightarrow \mathbb{R}$ such that $f = F$ outside of a set that can be made arbitrarily small. Several refinements of Lusin's theorem show that more regular functions f can be approximated by functions F with higher regularity, such as C^m maps.

A large part of analysis in Euclidean spaces may be generalized to Carnot groups, i.e. Lie groups whose Lie algebra admits a stratification. The goal of this talk is to provide results analogous to Lusin's theorem for functions whose domain or target is one of those groups.

The first part is devoted to the presentation of [1], where we give sufficient conditions under which an absolutely continuous horizontal curve $\gamma: [a, b] \rightarrow \mathbb{H}^1$ can be approximated by a C^m horizontal curve. Here with \mathbb{H}^1 we mean the classical Heisenberg group, which is the simplest example of non-commutative Lie group. We also show that we cannot weaken the regularity assumptions, proving that our result is optimal in the context of the Heisenberg group.

In the second part of the presentation we focus our attention on [2]. Here the functions are real valued as in the classical result, however the domain is a general Carnot group. We prove that k -approximate differentiability almost everywhere is equivalent to admitting a Lusin approximation by $C_{\mathbb{G}}^k$ maps. We also prove that existence of an approximate $(k - 1)$ -Taylor polynomial almost everywhere is equivalent to admitting Lusin approximation by maps in a suitable Lipschitz function space.

REFERENCES

- [1] Capolli, M., Pinamonti, A., Speight, G.: *A C^m Lusin Approximation Theorem for Horizontal Curves in the Heisenberg Group*, *Calculus of Variations and Partial Differential Equations* 60(49) (2021).
- [2] Capolli, M., Pinamonti, A., Speight, G.: *A C^k Lusin Approximation Theorem for Real-Valued Functions on Carnot Groups*, *ArXiv: 2107.12814* (2022).