SECOND-ORDER COVARIATION: AN ANALYSIS OF STUDENTS' FORMS OF REASONING AND TEACHER'S INTERVENTIONS WHEN MODELLING REAL PHENOMENA

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This research project aims to investigate covariational reasoning understood not only as the ability to visualize two or more magnitudes while co-varying simultaneously [1], but in a broader epistemological sense, as the ability to grasp relationships of invariance between mathematical objects. The need to better characterize more complex forms of reasoning performed by students in mathematical modelling activities led us to introduce second-order covariation, a form of covariation that consists in describing relations in which not only variables are involved but also parameters [2]. These enable to represent families of relationships between variables that is classes of real phenomena characterized, from a mathematical standpoint, through parameters, which determine the specificities of the mathematical model. The discussion of this theme arises not only from research needs in the field of Mathematics Education, i.e., the existence of a theoretical framework only partially useful to describe the covariational reasoning of students, but above all by its relevance in terms of teaching practices. There is a wide literature showing that in mathematical modelling situations the ability to reason covariationally is essential because it allows to envision the invariant relationships that exist between quantities involved in dynamic situations. The *Indicazioni Nazionali* for teaching mathematics in high schools [3] underline the relevance of introducing mathematical modelling as representation of classes of real phenomena. However, despite the acknowledged relevance of covariation for the learning of numerous mathematical concepts, in the Italian mathematics curriculum as well as in most textbooks, explicit references to this approach are generally absent. Teachers themselves have little specific knowledge of covariation and therefore struggle to introduce it into their teaching practices. Data analyzed in this study are from three teaching experiments conducted in the first grades of a scientific-oriented secondary school; their aim was the mathematical description of some real situations: the motion of a ball along an inclined plane [4] and the relationship between temperature and humidity described in the so-called psychrometric diagram. Using appropriate technological tools, students were guided in obtaining a mathematical formula describing such phenomena and in recognizing the different role played by variables and parameters in the writing and reading of different registers of mathematical representation. Students' reasoning processes and the evolution of the different semiotic aspects (spoken, gestural, representational) involved in the teaching-learning processes were analyzed; as well the support of technological tools and the role of the teacher in enhancing covariational reasoning through appropriate adaptive teaching strategies were considered. Eventually, this study led us to the elaboration of a broader theoretical framework about covariation.

References

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